Name: _____     Date: _____

## SINE/COSINE FUNCTIONS OF SPECIAL ANGLES: QUADRANT I
Memorize the following and be able to answer from memory in less than one minute in any order.

1. $\sin(0°) = \sin(0) = 0$

2. $\sin(30°) = \sin\left(\dfrac{\tau}{12}\right) = \dfrac{1}{2}$

3. $\sin(45°) = \sin\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$

4. $\sin(60°) = \sin\left(\dfrac{\tau}{6}\right) = \dfrac{\sqrt{3}}{2}$

5. $\sin(90°) = \sin\left(\dfrac{\tau}{4}\right) = 1$

6. $\cos(90°) = \cos\left(\dfrac{\tau}{4}\right) = 0$

7. $\cos(60°) = \cos\left(\dfrac{\tau}{6}\right) = \dfrac{1}{2}$

8. $\cos(45°) = \cos\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$

9. $\cos(30°) = \cos\left(\dfrac{\tau}{12}\right) = \dfrac{\sqrt{3}}{2}$

10. $\cos(0°) = \cos(0) = 1$

## PRACTICE I

1. $\cos(45°) =$

2. $\sin(30°) =$

3. $\sin(45°) =$

4. $\cos(60°) =$

5. $\sin(90°) =$

6. $\cos(0°) =$

7. $\cos(30°) =$

8. $\cos(90°) =$

9. $\sin(0°) =$

10. $\sin(60°) =$

## PRACTICE II

1. $\cos\left(\dfrac{\tau}{6}\right) =$

2. $\sin\left(\dfrac{\tau}{6}\right) =$

3. $\cos\left(\dfrac{\tau}{4}\right) =$

4. $\sin\left(\dfrac{\tau}{4}\right) =$

5. $\cos(0) =$

6. $\cos\left(\dfrac{\tau}{12}\right) =$

7. $\sin\left(\dfrac{\tau}{8}\right) =$

8. $\cos\left(\dfrac{\tau}{8}\right) =$

9. $\sin\left(\dfrac{\tau}{12}\right) =$

10. $\sin(0) =$

## PRACTICE III

1. $\sin\left(\dfrac{\tau}{4}\right) =$

2. $\cos\left(\dfrac{\tau}{8}\right) =$

3. $\sin\left(\dfrac{\tau}{6}\right) =$

4. $\cos\left(\dfrac{\tau}{4}\right) =$

5. $\sin(0) =$

6. $\cos\left(\dfrac{\tau}{12}\right) =$

7. $\cos\left(\dfrac{\tau}{6}\right) =$

8. $\cos(0) =$

10. $\sin\left(\dfrac{\tau}{8}\right) =$

9. $\sin\left(\dfrac{\tau}{12}\right) =$

## ANGLES WITH SPECIAL REFERENCE ANGLES

| Quadrant I | Quadrant II | Quadrant III | Quadrant IV |
|:---:|:---:|:---:|:---:|
| $\alpha$ | $180° - \alpha$ | $180° + \alpha$ | $360° - \alpha$ |
| $30°$ | $150°$ | $210°$ | $330°$ |
| $45°$ | $135°$ | $225°$ | $315°$ |
| $60°$ | $120°$ | $240°$ | $300°$ |
| $\alpha$ | $\dfrac{\tau}{2} - \alpha$ | $\dfrac{\tau}{2} + \alpha$ | $\tau - \alpha$ |
| $\dfrac{\tau}{12}$ | $\dfrac{6\tau}{12} - \dfrac{\tau}{12} = \dfrac{5\tau}{12}$ | $\dfrac{6\tau}{12} + \dfrac{\tau}{12} = \dfrac{7\tau}{12}$ | $\dfrac{12\tau}{12} - \dfrac{\tau}{12} = \dfrac{11\tau}{12}$ |
| $\dfrac{\tau}{8}$ | $\dfrac{4\tau}{8} - \dfrac{\tau}{8} = \dfrac{3\tau}{8}$ | $\dfrac{4\tau}{8} + \dfrac{\tau}{8} = \dfrac{5\tau}{8}$ | $\dfrac{8\tau}{8} - \dfrac{\tau}{8} = \dfrac{7\tau}{8}$ |
| $\dfrac{\tau}{6}$ | $\dfrac{3\tau}{6} - \dfrac{\tau}{6} = \dfrac{2\tau}{6} = \dfrac{\tau}{3}$ | $\dfrac{3\tau}{6} + \dfrac{\tau}{6} = \dfrac{4\tau}{6} = \dfrac{2\tau}{3}$ | $\dfrac{6\tau}{6} - \dfrac{\tau}{6} = \dfrac{5\tau}{6}$ |

## SINE/COSINE FUNCTIONS OF SPECIAL ANGLES: ALL QUADRANTS

Use the memorized trig values from the first quadrant and the techniques explained in this section to calculate the sine or cosine of any angle with a special reference angle. Practice until you can do 10 of these problems in less than 3 minutes.

*Note that the sine function is positive in the first and second quadrants. Cosine is positive in the first and fourth.*

1. $\sin\left(\dfrac{\tau}{12}\right) = \dfrac{1}{2}$ (QI)

2. $\sin\left(\dfrac{5\tau}{12}\right) = \sin\left(\dfrac{\tau}{12}\right) = \dfrac{1}{2}$ (QII)

3. $\sin\left(\dfrac{7\tau}{12}\right) = -\sin\left(\dfrac{\tau}{12}\right) = -\dfrac{1}{2}$ (QIII)

4. $\sin\left(\dfrac{11\tau}{12}\right) = -\sin\left(\dfrac{\tau}{12}\right) = -\dfrac{1}{2}$ (QIV)

5. $\cos\left(\dfrac{\tau}{12}\right) = \dfrac{\sqrt{3}}{2}$ (QI)

6. $\cos\left(\dfrac{5\tau}{12}\right) = -\cos\left(\dfrac{\tau}{12}\right) = -\dfrac{\sqrt{3}}{2}$ (QII)

7. $\cos\left(\dfrac{7\tau}{12}\right) = -\cos\left(\dfrac{\tau}{12}\right) = -\dfrac{\sqrt{3}}{2}$ (QIII)

8. $\cos\left(\dfrac{11\tau}{12}\right) = \cos\left(\dfrac{\tau}{12}\right) = \dfrac{\sqrt{3}}{2}$ (QIV)

1. $\sin\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$ (QI)

2. $\sin\left(\dfrac{3\tau}{8}\right) = \sin\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$ (QII)

3. $\sin\left(\dfrac{5\tau}{8}\right) = -\sin\left(\dfrac{\tau}{8}\right) = -\dfrac{\sqrt{2}}{2}$ (QIII)

4. $\sin\left(\dfrac{7\tau}{8}\right) = -\sin\left(\dfrac{\tau}{8}\right) = -\dfrac{\sqrt{2}}{2}$ (QIV)

5. $\cos\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$ (QI)

6. $\cos\left(\dfrac{3\tau}{8}\right) = -\cos\left(\dfrac{\tau}{8}\right) = -\dfrac{\sqrt{2}}{2}$ (QII)

7. $\cos\left(\dfrac{5\tau}{8}\right) = -\cos\left(\dfrac{\tau}{8}\right) = -\dfrac{\sqrt{2}}{2}$ (QIII)

8. $\cos\left(\dfrac{7\tau}{8}\right) = \cos\left(\dfrac{\tau}{8}\right) = \dfrac{\sqrt{2}}{2}$ (QIV)

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1. $\sin\left(\dfrac{\tau}{6}\right) = \dfrac{\sqrt{3}}{2}$ (QI)

2. $\sin\left(\dfrac{\tau}{3}\right) = \sin\left(\dfrac{2\tau}{6}\right) = \sin\left(\dfrac{\tau}{6}\right) = \dfrac{\sqrt{3}}{2}$ (QII)

3. $\sin\left(\dfrac{2\tau}{3}\right) = \sin\left(\dfrac{4\tau}{6}\right) = -\sin\left(\dfrac{\tau}{6}\right) = -\dfrac{\sqrt{3}}{2}$ (QIII)

4. $\sin\left(\dfrac{5\tau}{6}\right) = -\sin\left(\dfrac{\tau}{6}\right) = -\dfrac{\sqrt{3}}{2}$ (QIV)

5. $\cos\left(\dfrac{\tau}{6}\right) = \dfrac{1}{2}$ (QI)

6. $\cos\left(\dfrac{\tau}{3}\right) = \cos\left(\dfrac{2\tau}{6}\right) = -\cos\left(\dfrac{\tau}{6}\right) = -\dfrac{1}{2}$ (QII)

7. $\cos\left(\dfrac{2\tau}{3}\right) = \cos\left(\dfrac{4\tau}{6}\right) = -\cos\left(\dfrac{\tau}{6}\right) = -\dfrac{1}{2}$ (QIII)

8. $\cos\left(\dfrac{5\tau}{6}\right) = \cos\left(\dfrac{\tau}{6}\right) = \dfrac{1}{2}$ (QIV)

**PRACTICE IV**

1. $\sin\left(\dfrac{3\tau}{8}\right) =$

2. $\cos\left(\dfrac{\tau}{3}\right) =$

3. $\sin\left(\dfrac{5\tau}{6}\right) =$

4. $\cos\left(\dfrac{7\tau}{8}\right) =$

5. $\cos\left(\dfrac{2\tau}{3}\right) =$

6. $\cos\left(\dfrac{7\tau}{12}\right) =$

7. $\sin\left(\dfrac{2\tau}{3}\right) =$

8. $\cos\left(\dfrac{5\tau}{8}\right) =$

9. $\sin\left(\dfrac{5\tau}{12}\right) =$

10. $\sin\left(\dfrac{5\tau}{8}\right) =$

**PRACTICE V**

1. $\sin\left(\dfrac{5\tau}{12}\right) =$

2. $\cos\left(\dfrac{5\tau}{6}\right) =$

3. $\sin\left(\dfrac{5\tau}{8}\right) =$

4. $\cos\left(\dfrac{7\tau}{12}\right) =$

5. $\cos\left(\dfrac{7\tau}{8}\right) =$

6. $\cos\left(\dfrac{11\tau}{12}\right) =$

7. $\sin\left(\dfrac{\tau}{3}\right) =$

8. $\cos\left(\dfrac{3\tau}{8}\right) =$

9. $\sin\left(\dfrac{11\tau}{12}\right) =$

10. $\sin\left(\dfrac{7\tau}{8}\right) =$

## PRACTICE VI

1. $\sin\left(135°\right) =$

2. $\cos\left(150°\right) =$

3. $\sin\left(225°\right) =$

4. $\cos\left(240°\right) =$

5. $\cos\left(315°\right) =$

6. $\cos\left(225°\right) =$

7. $\sin\left(315°\right) =$

8. $\cos\left(300°\right) =$

9. $\sin\left(210°\right) =$

10. $\sin\left(120°\right) =$

## TANGENT/COTANGENT/SECANT/COSECANT FUNCTIONS OF ANGLES WITH SPECIAL REFERENCE ANGLES: ALL QUADRANTS

Convert tangent, cotangent, secant, and cosecant into functions of sine and cosine to calculate. Practice until you can do 10 of these problems in less than 5 minutes.

$$\tan\left(\theta\right) = \frac{\sin\left(\theta\right)}{\cos\left(\theta\right)}, \quad \sec\left(\theta\right) = \frac{1}{\cos\left(\theta\right)}, \quad \csc\left(\theta\right) = \frac{1}{\sin\left(\theta\right)}$$

*Note that the tangent is positive in the first and third quadrant. Cotangent, the reciprocal of tangent, is likewise positive in the first and third quadrants. Secant, like its reciprocal cosine, is positive in the first and fourth quadrants. Cosecant, like its reciprocal sine, is positive in the first and second quadrants.*

1. $\tan\left(\dfrac{5\tau}{12}\right) = -\tan\left(\dfrac{\tau}{12}\right) = -\dfrac{\sin\left(\frac{\tau}{12}\right)}{\cos\left(\frac{\tau}{12}\right)} = -\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$ (QII)

2. $\sec\left(\dfrac{5\tau}{8}\right) = -\sec\left(\dfrac{\tau}{8}\right) = -\dfrac{1}{\cos\left(\frac{\tau}{8}\right)} = -\dfrac{1}{\frac{\sqrt{2}}{2}} = -\dfrac{2}{\sqrt{2}} = -\sqrt{2}$ (QII)

3. $\csc\left(\dfrac{2\tau}{3}\right) = -\csc\left(\dfrac{\tau}{6}\right) = -\dfrac{1}{\sin\left(\frac{\tau}{6}\right)} = -\dfrac{1}{\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$ (QIII)

4. $\cot\left(\dfrac{2\tau}{3}\right) = \cot\left(\dfrac{\tau}{6}\right) = \dfrac{\cos\left(\frac{\tau}{6}\right)}{\sin\left(\frac{\tau}{6}\right)} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ (QIII)

5. $\sec\left(\dfrac{11\tau}{12}\right) = \sec\left(\dfrac{\tau}{12}\right) = \dfrac{1}{\cos\left(\frac{\tau}{12}\right)} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$ (QIV)

6. $\tan\left(\dfrac{5\tau}{8}\right) = \tan\left(\dfrac{\tau}{8}\right) = \dfrac{\sin\left(\frac{\tau}{8}\right)}{\cos\left(\frac{\tau}{8}\right)} = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$ (QIII)

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## PRACTICE VII

1. $\csc\left(\dfrac{5\tau}{12}\right) =$

2. $\sec\left(\dfrac{5\tau}{6}\right) =$

3. $\tan\left(\dfrac{5\tau}{8}\right) =$

4. $\tan\left(\dfrac{7\tau}{12}\right) =$

5. $\cot\left(\dfrac{7\tau}{8}\right) =$

6. $\csc\left(\dfrac{11\tau}{12}\right) =$

7. $\sec\left(\dfrac{\tau}{3}\right) =$

8. $\csc\left(\dfrac{3\tau}{8}\right) =$

9. $\tan\left(\dfrac{11\tau}{12}\right) =$

10. $\csc\left(\dfrac{7\tau}{8}\right) =$

## PRACTICE VIII

1. $\tan\left(\dfrac{3\tau}{8}\right) =$

2. $\cot\left(\dfrac{\tau}{3}\right) =$

3. $\sec\left(\dfrac{5\tau}{6}\right) =$

4. $\csc\left(\dfrac{7\tau}{8}\right) =$

5. $\tan\left(\dfrac{2\tau}{3}\right) =$

6. $\sec\left(\dfrac{7\tau}{12}\right) =$

7. $\csc\left(\dfrac{2\tau}{3}\right) =$

8. $\cot\left(\dfrac{5\tau}{8}\right) =$

9. $\csc\left(\dfrac{5\tau}{12}\right) =$

10. $\sec\left(\dfrac{5\tau}{8}\right) =$

## PRACTICE IX

1. $\sec\left(135°\right) =$

2. $\tan\left(150°\right) =$

3. $\cot\left(225°\right) =$

4. $\csc\left(240°\right) =$

5. $\cot\left(315°\right) =$

6. $\tan\left(225°\right) =$

7. $\csc\left(315°\right) =$

8. $\sec\left(300°\right) =$

9. $\csc\left(210°\right) =$

10. $\cot\left(120°\right) =$